

STRENGTH IN NUMBERS

For most teenagers, being condemned to spend the summer doing math problems would be a special circle of hell. But for the gifted teens SIOBHAN ROBERTS meets at the Canada/USA Mathcamp, it's paradise -- five weeks with 110 other nerds who love math's 'elegance, its beauty, its innumerable coincidences'

Siobhan Roberts,
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AT 9 O'CLOCK ON A SATURDAY MORNING in the final week of Mathcamp, only a small number of the 110 campers have made it to the cafeteria for breakfast. Over the past five weeks, they have been staying up later and later, for the all-nighter bridge tournament, the 50-hour puzzle hunt, the last-minute yearbook edits or simply to do more math. "Meet here at midnight Tuesday night" is posted on the common-room wall under the heading, "Math Until We Die."

Those campers who have made it out of bed shovel scrambled eggs down the hatch -- one, seeking optimal efficiency, lowers his mouth to the same horizontal plane as his plate -- and another day at Mathcamp has begun.

"We basically do whatever we want every day," says Mathieu Guay-Paquet, a lanky 18-year-old camper from Montreal, with curly helixes of brown hair dangling at his shoulders. "There's a lot of freedom and no curfew . . . because if we had to go to bed at a certain time, that might interrupt some important mathematical ideas."

In fact, though, there are a few curbs on nocturnal activity. By 10 p.m., campers must sign in to the common room, which by that hour, in the middle of summer, is a hotbed not only of number theory but of very odorous teenagers sweating themselves silly at killer table tennis, chess and Rock Paper Scissors. Campers have to be out of the halls by midnight, but they can gather in their dorm rooms to

do what they wish ad infinitum, or at least to the limits of honest computation.

The camper who organized the bridge tournament, Frank Sun, 18, from Vancouver, staggers into the cafeteria, and soon enough he is gape-jawed and asleep over his breakfast tray. The devotees pass him without a second glance as they disperse and head to their classrooms across campus.

The Canada/USA Mathcamp, as it is officially called, is located this year on the University of Puget Sound, in Tacoma, Wash. (The location has alternated between U.S. and Canadian campuses since the camp started in 1993; Canadians usually make up about 25 per cent of the group.) UPS is a late-19th-century, ivy-covered setting, with plenty of nooks in which campers can seclude themselves -- in the rose garden by the parabolic spouting fountain, in the shade of towering and well-manicured elliptical bushes, on the sprawling staircases, the tiered slopes of which are used as makeshift desktops.

I first made contact with Mathieu by e-mail, his preferred medium, while he was still at home packing. His e-mail prefix, "mythical_mat," fits his talents. "When Mathieu was 2," recalled his mother, Gisele Guay, "he wanted to know how absolutely everything worked. I thought for sure he was going to be a mechanic. It wasn't until he went to school that we realized he would be a math wizard." His father, Gaetan Paquet, added that when Mat was 8 he asked his parents (who are both computer consultants) for a postsecondary math textbook that he's still using today.

This is Mat's second year at Mathcamp, and the second arithmetic program he has attended this summer (he also works at A&W, the only place that would hire a kid who stipulated he needed more than five weeks off). In total, he has been to six in the past three years, including Canada's National Math Camp and several of the

provincial camps operated by the Canadian Mathematical Society. At the CMS, sessions are more regimented and only one week in length, and campers are invited according to their showing in the national Open Mathematics Challenge. The Canada/USA Mathcamp recruits at high schools across North America and internationally, like most of the more than 25 national and state math camps in the United States.

This camp is by far Mat's favourite, for its philosophy of giving campers the freedom to do all the math their hearts desire. It is safe to say math consumes Mat, though he also enjoys computers (especially programming), piano (though he doesn't practise enough), racquet sports, reading, card games and tutoring.

"I don't actually learn anything in math class. I go to socialize," Mat's mother recently heard him say. He is heading into his second year of CEGEP, after which he plans to study pure mathematics at the University of Waterloo.

One of my first questions for Mat was whether he takes any time off from math while school is out. "I don't think I ever really take a break," he wrote. "When you still wonder about coincidences and other everyday things, there is an endless supply of questions to ask. What are the equations governing this particular movement? How can you find a winning strategy to a game? What are the properties of this shape, or that one? Whenever I'm not doing anything in particular, I usually just let my mind wander, and my line of thought most often brings me to a question about the mathematics of everyday life around me.

"That being said," he continued, "you might be wondering what exactly I have been pondering this last month. I have tried to derive Cardano's formula, the general solution to a third-degree polynomial (i.e., solving $x^3 + bx^2 + cx + d = 0$), and haven't got anywhere yet."

He was also working on "inventing a generalization of calculus, which I call semi-calculus, which would enable someone to compute the half-derivative of a function." He called this "a dream I've had for a long time."

What did his friends think of the fact that he willingly spends almost all his lazy summer days this way? They understand that he loves math, he said, but they don't know much more. "While I can't easily explain it to the layperson, I always assure them that it has almost nothing to do with the boring math we do in school."

And what, exactly, would he do at Mathcamp? Anything was possible, he said. Different courses are offered each week of camp, with the last week's subjects chosen by a vote. Each day is divided into six class sessions, with a choice of four subjects of varying difficulty in each timeslot. But his itinerary from last year gave a good indication. It included a five-week course in Real Analysis, three-week courses in Linear Algebra and Math of Communication, one-week courses in the Math of Juggling, Calculus without Calculus, Matrix Mechanics, Order of Magnitude Physics, Basic Cryptography, Numerical Methods and, surprisingly, Media ("reading between the li(n)es"), and such one-shot classes as Big Numbers ("these were truly huge numbers").

"Are you beginning to see a pattern here?" he asked. "I also did some geometric crochet, forming a woollen model of the hyperbolic plane. I participated in the math relays and in the team problem-solving competition. I played cards an insane number of times, and I learned how to play bridge. I went hiking. I went white-water rafting. I gave an evening class on how to take square roots by hand, the long division way. I stayed up late with friends to discuss philosophy, math, life experience and other matters. I read Douglas Hofstadter's amazing book, Gödel, Escher, Bach: An Eternal Golden Braid. I generally had a blast of a good time."

This year, he concluded, "I plan to have a blast of a good time again."

Math, of course, is not everyone's idea of a blast. Most of society still cringes from the fear instilled by memorizing public-school multiplication tables and fumbling around with protractors, and has happily left behind all things mathematical.

Mathcamp is the negative reciprocal of all that. There is no lake on which to water-ski, not a horse to ride, not even rocks to collect and paste on particleboard. Founded in 1993 by George Thomas, a mathematician based in London, Ont., Mathcamp was not created to provide a summer wonderland in the woods, but to be an incubator where gifted students aged 15 to 18 could pursue their beloved science beyond the bounds of outdated and unimaginative curricula, not to mention burnt-out teachers.

"Mathcamp is like standing on a mountaintop and viewing the terrain below," says Mr. Thomas, rhapsodizing a bit on his creation. (Last year, he abdicated as director here to start another camp, MathPath, for children 12 to 14.) "In these environments, you can see the lay of the land and all the possibilities of mathematics stretching out before you to the horizon. Math in school is like sitting on the sidewalk in the middle of the city," he says. "All you can see are the few buildings right in front of you."

For me, a mathematical dilettante, this notion of beauty is what makes mathematicians so alluring. Although I have no formal credentials (other than the fact that I skipped Grade 7 math, to my detriment), I find myself writing the biography of the late Donald Coxeter, twice a guest lecturer at Mathcamp. He was introduced to me as the greatest living classical geometer of the last century, the man who saved classical geometry from near extinction. I was hooked. And I was

similarly entranced when early in my research I asked him why he did what he did. "Nobody asks an artist why they do what they do," he said. "I'm like any other artist. My passion just happens to be shapes and numbers."

For Coxeter, mathematics held the same visceral pull as a breathtaking pianissimo in Mozart or the palette of threatening clouds in a Vermeer. It had to do with its elegance, and the elation of discovery that comes after an inordinate amount of fifth-gear thinking.

"Math is an art in some sense, but I would also say it is a struggle," says David Savitt, 25, a Mathcamp '96 alumnus, now the deputy director of the camp and a professor at McGill University. He received his PhD in mathematics from Harvard at 22; most of the camp's 20 mentors and faculty are similarly qualified or well on their way.

"These things that we are thinking about are hard to understand," even Prof. Savitt admits. "They are mind-boggling. It's the craziest battle to wrap your mind around these extremely abstract ideas. But the joy comes from finally understanding something that you didn't understand before. It's a phenomenal feeling even to spot subtle connections. You have to smile when it happens."

The morning I tracked Mat down at Mathcamp, the selection of 9 a.m. classes was Hardcore Problem Solving, Algebraic Geometry, Math Jewels, Algorithms, and a subject in the flesh that truly gets campers awe-inspired: renowned Princeton mathematician John Conway.

This was the last day of Prof. Conway's week at camp and it was a triple header: a lecture in the morning, a math-game challenge with the campers at noon, and a finale lecture that evening. On first sighting, he looks the part of the math deity, a

bearded and portly Socrates with quick and playful eyes that belie his 65 years. But his collection of T-shirts and yesterday's shorts camouflage him well among the campers. He returns to Mathcamp nearly every year.

"There are not many professional mathematicians who retain an interest in childish things," Prof. Conway says proudly. (Besides his spiritual descendants here at Mathcamp, he has four adult daughters in his native England, and on this side of the Channel, two teenaged boys as well as a two-year old son he watches two days a week, each grouping from a subsequent marriage.) In addition to his playfulness, though, Prof. Conway does possess a seriously rigorous mathematical mind. He was wooed to Princeton from Cambridge in 1985, at the height of his fame for discovering surreal numbers, an entirely new class of numbers that approach the infinite and the infinitesimal -- smaller than any positive real number and yet greater than zero. What made him a more popular legend, though, were his more recreational inventions, from his famous Game of Life in the 1970s to Philosopher's Football and the Mathcamp preoccupation, Dots and Boxes. (See sidebar.) "Every now and then I do a very interesting piece of math, though not as often as I used to," Prof. Conway says. "Most of the time I'm thinking of trivial math. But when I hit the white hot stuff, it certainly is fabulous -- when you find something that is deep and subtle and no one has ever happened on it before. When I found the world of surreal numbers in the 1970s, I was walking around in a daydream, happily exploring this world in my head for six weeks."

In these parts, Prof. Conway is a wizard and a saint, the living embodiment of math for math's sake. "Application is not the point," he says, even though as a consultant in the 1980s, he advised Bell Laboratories on a piece of satellite technology (using the theory "eight-dimensional sphere packing") that Bell patented and made millions.

"Beautiful intellectualizing, that is the satisfaction," he says. "The intellectual life of society should allow some people to do interesting things."

At the beginning of his morning lecture, the amphitheatre is about half full with 30 or so campers, a few still straggling in barefoot and barely awake. The chatter is that he will be performing his infamous tangle trick, as he did last year. But first he opens the floor to ideas for what he should discuss at his finale talk that evening. Three campers in a row raise their hands to request the Game of Life. "I've got a better idea," he says.

He turns to the blackboard and scrawls his chosen topic: "How to Beat Children at Their Own Games."

That settled, Prof. Conway begins the lecture. He pulls one jump rope out of his pocket and another from a plastic bag, and selects four volunteers. He is, indeed, doing his notorious tangle trick. A tangle, as he defines it, is "a piece of knottiness with four ends coming out."

He warns anyone who saw the trick last year to keep their "big mouths shut." Mat, who has seen it before, assumes his characteristic Thinker's pose, saying, "It'll be a lot of fun anyway."

The four volunteers stand across from one another in the formation of a rectangle, each holding one end of the skipping ropes to form the long top and base. From there Prof. Conway shouts his "square-dancing calls": Either "Twist it up!" indicating that the campers should each rotate around one position, or "Turn 'em 'round!" meaning the camper in the top right corner of the rectangle should cross under and become the bottom right corner.

"TWIST!" He pauses, waiting for his volunteers to obey. "TWIST again! TWIST! . . . TURN! . . . TWIST!" The square dancers move hesitantly, waylaid by giggles and general uncertainty. The movements are recorded mathematically on the blackboard: 0, 1, 2, 3, $-1/3$, $2/3$. . . It was one of Conway's discoveries as a teenager that tangles have the same structure as fractions -- for each twist add one, and for each turn take the negative reciprocal.

Soon enough the ropes are in a fine mess of a tangle. Conway then threads a plastic bag over them to conceal the knot, and commands, "Now, have a go at getting it back to zero [untangled]."

Silence.

"Come on now, way you go!"

If the mathematical map on the blackboard is correct, it will allow the tanglers to retrace their steps, like following a string out of a maze. After about 20 minutes, and a few erroneous turns and twists that draw shouts from the audience, they declare themselves unknotted.

"Well, I'm not so sure," Prof. Conway says. "I have a horrible feeling someone twisted when they should have turned." He rips off the plastic bag. Each camper yanks on their end of the rope and, to great applause, the tangle dissolves.

Prof. Conway goes on to describe the theory behind the trick. "I've been struggling with this problem about knots all my life: How to describe a knot to someone over the telephone. Now, keep in mind," he adds, "that the person on the other end of the line is probably a nerd just like you . . ."

The demographics of Mathcamp are what you might expect. Frank Sun happily admits he is known as a math geek at school -- his classmates and teachers have called him "Gretzky" ever since he scored 99 per cent on his provincial math exams. He is off to Simon Fraser University on full scholarship this fall, for a combined major in computing science, molecular biology and biochemistry. He wants to be a surgeon. "I don't want to do math professionally," he says, smiling ear-to-ear. "Math is just for fun."

Anna de Bakker from Winnipeg is teased about her academic prowess even during her infrequent phone calls home. "My brother was helping my mom clean my room and he said it smelled of rotten algebra and dirty geometry."

Anna is a 5-foot-6, slumped-shouldered 14-year-old who usually sports groovy plum suede Skechers shoes and cargo pants. She also sings (she's joined the camp a capella group, the Contrapositiones, who will perform at the closing talent show) and only recently gave up the cello. She is fluent not only in English and the universal language of mathematics, but also in French, Dutch, German, Latin and Spanish. One of her brothers (she is the youngest of five siblings) used to joke that she was someone who could read War and Peace in five languages but couldn't tie her shoes (she plans to hone her Latin in Grade 10 next year at St. Paul's, an august New England prep school to which she's won full scholarship).

"We always thought that she would go into languages," says her father, Willem de Bakker, an engineering consultant at Manitoba Hydro. "But then out of the blue she came home from school with the application for Mathcamp and she worked like crazy on the quiz to qualify. That was the first inkling we had that she was as good as she is with math."

The application process deliberately discourages meddling mothers and fathers

who want either to unload their hyperdimensional teenagers for more than a month or to mould their straight-A prodigies into the next Bill Gates. Tuition is a steep \$2,995 (U.S.), made much more affordable with a sliding scale of financial aid, scholarships and fundraising advice. And campers themselves have to want to come: Each must write an essay explaining their motivations and what they expect to get out of Mathcamp.

They submit two letters of recommendation, but no grades. Instead, they must attempt 10 problems such as the following: "Everyday at Mathcamp, Frank and Lilian competed to see who could solve more problems in problem-solving class. Each day at least one of them was able to do more than half the problems. On days when Frank solved more than half the problems, he won twice as often as he lost. On days when Lilian solved more than half the problems, she won five times as often as she lost. There were 25 classes in total. On how many days did Lilian win? You can assume there were no ties."

This is the sort of mind-numbing exercise that made me perform dismally poorly on the LSAT. Twice. The kind that separates the world's rigorous intellectuals from those who are doomed to stand on the sidelines and cheer.

In fact, there is also a cheerleading camp taking place at UPS this summer, populated by the diametric opposites of the Mathcampers. The cheerleaders blow-dry and beribbon their hair at 6 a.m., apply and reapply moisturizer to their head-to-toe perfect tans, and bound around the campus like baby deer, wearing pompommed flip-flops and very short-shorts (which eventually give way, on the day team portraits are taken, to perfectly creased white tutus).

Mathcampers, meanwhile, don't always wash or even brush their hair. Quite a number sport glasses (Mat's are for his myopia), they lug knapsacks everywhere,

wear their keys around their necks and can be spotted, on occasion, wearing socks with their sandals.

There are only 20 male cheerleaders among 500 campers, while at Mathcamp, boys outnumber girls at a ratio of 5:1. (Racial variety is also difficult to achieve at Mathcamp, with Caucasian and Asian backgrounds exponentially higher than the rest, though the camp tries for diversity with preferential scholarships and aid). You would think it might be hard for a mathematician to concentrate with cheerleaders swarming the field outside their classrooms, highkicking to a megaphoned "and 1 and 2, up, 3 and 4, and 5 and 6 and 7 and 8!" in a highly unfeminist routine. But Mathcampers hardly seem to notice, much less lust after their neighbours.

The nerd-to-hip correlation, however, as one counsellor pointed out, is all relative. Last summer, for example, the Mathcampers shared facilities with Super Camp, a self-esteem boot camp. At the end of the summer, Super Campers made T-shirts declaring "No FIMAGE"-- with FIMAGE (Fear of Image) in a red crossed-out circle, as on a "No Parking" sign -- but found few buyers among themselves. Mathcampers loved the shirts, bought them and wore them everywhere.

They may be nerds, but they are self-loving nerds, with a healthy sense of humour. The bulletin board in the lounge is full of joke notices like "Summer Camp Cross Registration":

Due to the overwhelming popularity of some of the other camps here at UPS, we have decided to set up a cross-registration program between the camps. Note that cross registration goes both ways, so to accommodate for additional campers taking our classes, we have decided to add the following:

How to say "No" to peer pressure (for SuperCampers)

That IS a Pencil in My Pocket (for "Abstinence Camp," the two nuns recently spotted in the rose garden)

How to Cheer in a Non-Sexual Manner (for the cheerleaders)

A sign-up sheet solicited suggestions for this year's Mathcamp T-shirt slogan. The time-honoured T-shirt slogan, "A mathematician is cautious in the presence of the obvious," was met with a scribbled "YAWN!" Some of the alternatives:

Math: Because when you can't get laid, you might as well be good at something.

"A proof is a proof. What kind of proof? It's a proof! A proof is a proof and when you have a good proof it's because it's proven." -- Prime Minister Jean Chrétien

Nothing comes between me and my Mathcamp.

I have a really great slogan, but there's not enough room on this T-shirt. (A play on French mathematician Pierre de Fermat's infamous note on the manuscript of his last theorem: "I have a truly marvellous demonstration of this proposition which this margin is too narrow to contain." As Mat remarks later, "This has become a joke in mathematics, as the missing proof took more than 300 years to find.")

The winning choice is a little meatier: "In mathematics, existence is freedom from contradiction."

Yet another posting announces the big event in the cafeteria at lunch: The Mathcampers Versus John Conway Challenge: "We've studied . . . we've trained . .

. now we're ready to challenge Conway to a game of Dots and Boxes! Come watch us play (and probably lose)."

The camp Dots and Boxes team has been practising for days with Prof. Conway's book on the subject, *Winning Ways For Your Mathematical Plays* (with Richard Guy and Elwyn Berlekamp). The game is played between two players on a rectangular array of dots, such as a 4-by-3 grid. The players take turns joining two horizontally or vertically adjacent dots. When players complete the fourth side of a box they initial it, and score a point. The winner is the player with the most boxes when the grid is full. (See sidebar.)

The Dots-and-Boxers were horrified to find out that Prof. Conway himself borrowed his book from the camp office last night for a refresher. The team leader and camp director both ran after him in protest. He agreed to memorize only one page: "I hope that's enough to beat the kiddies. I haven't played the game seriously in ages."

The kids figure the only way they can beat Prof. Conway is to force him to play their entire team simultaneously. The master agrees and begins circling the two combined lunch tables at which his dozen opponents are seated. He makes his first move with each challenger and continues around the circuit. It's not long before many are giving up. As Prof. Conway thins out the competition, the crowd thickens, egging on fellow campers, one of whom is balancing a spoon on his nose while he plays.

"I'm trying to psych him out," says Brendan Flaherty, from Rhode Island. It works.

"Wait a minute," says Prof. Conway next time around. "What's happened here? You seem to have won!"

Prof. Conway loses not only that game, but three others. It's an event that will go down in Mathcamp history, as only one other camper has ever succeeded in conquering John Conway. His disclaimer comes at his finale lecture that evening: "You appreciate, of course," he says to the packed house, "that even if you succeeding in beating me at Dots and Boxes, you didn't really -- it was just that I made a mistake."

Along with the complex math in his lecture, he offers a few homilies to guide his young followers' mathematical futures: Figuring out how to win games is fun in itself, but looking for the larger theories behind games leads to broader understanding; "Stay interested in everything" (his other interests include astronomy and etymology); "Take it as axiomatic that you are stupid. If you think you have proved something, think again"; and finally, "If you have indeed discovered something, but then discover that someone else discovered it before you, consider yourself in good company, and mark your progress. If you find something already discovered 2,000 years ago, then 200, then 20, then 2, at least you are improving. And then, if you're lucky, next maybe you'll discover something new."

Afterward, he is cornered at the pop machine and bombarded with questions. How does he spend his days? Playing, he says, adding in my direction, "It would be horrible to have a job where you have to work." They trail him back to the common room, queries flying. Finally, he arrives and is persuaded to repeat a few of the tricks he has performed over the week, which make him seem as much magician as mathematician: spinning a bent-out coat hanger over his head, for example, with a penny balanced on the tip of the hook.

Then he plunks himself down on a couch. He accepts one final challenger for Dots

and Boxes, at the same time he and Mat start a game of Philosopher's Football. After an hour and a half, at 11:30 p.m., the camper to Prof. Conway's left has conked out, flopped over and asleep on the arm of the couch. The match of Philosopher's Football, meanwhile, is taking forever.

"He's giving me a sweaty game," Prof. Conway says. "Who," he asks Mat, "have you been playing with?"

"No one," Mat says, "since you taught me how to play last year."

At that moment, I get Mathcamp: Since last summer, Mat hasn't been able to find anybody who knew this game, or wanted to play. What Mathcamp provides is an instant peer group -- people who laugh at math jokes, play math games and entertain obsessions that seem weird to friends at home. It is a haven, an escape, for kids who relate better to mathematics than to people (which is not so true of Mat, Anna or Frank, but is for some of the more stereotypical fellow campers). As one confessed on the end-of-camp survey: "Now there is only 330 days of a dream to pass by, and then we can come back to the real world, to Mathcamp."

"Curses," says Prof. Conway, realizing he might lose this game as well. "I might cry about that move." And indeed, Mat prevails, though the professor, overconfidently, gave himself a handicap at the outset.

"I wish I wasn't going home tomorrow," Prof. Conway says. "We really should play again."

With his mentor gone, it comes as a shock to Mat to realize Mathcamp is about to end. He says everyone is trying "to cram in as much as possible in these last moments together, going to plenty of classes and posting and participating in other

activities." Friday night is the talent show, with the Contrapositiones, as well as juggling, tap dancing, poetry, Irish dancing and a comedy skit called The House With Too Many Perpendiculars. In closing, all the campers stand and sing this year's Mathcamp anthem.

The final evening, Saturday, brings the traditional too-sad-to-sleep all-nighter. The activity board is repartitioned into hours and filled up with impromptu activities. There's packing, cleaning and room inspection, and the "Future of You" talk, where staff answer questions about college. And of course, more math: The "30 Proofs in 30 Minutes" event, and a 12-star class taught by three teachers at once (the previous maximum level of difficulty was four stars). There's soccer and computer-game tournaments, and above all, yearbook signing, pictures taken and goodbyes said before shuttle buses subtract people from camp five and six at a time, and take them to the airport.

"There were fewer activities as the sad time of parting came near," Mat recounts for me after he gets home. "The first shuttle left at 3 a.m. Sunday, but I was among the last to leave. When my shuttle left at 6 p.m., there were only five of us campers left."

Even my vicarious, dilettante's stint at Mathcamp left a lasting impression. For one thing, I learned how to tango (surprise, surprise, Mat also has great rhythm on the dance floor). And I witnessed in action the answer to one of the first questions I had put to Mat: Why did he love math so much?

"Gosh," he wrote, "that's a hard question.

"I love it for its elegance, its beauty, its innumerable coincidences. I love math because it is one of the only things in this life in which there seems to be certainty

and truth. I love math because it is an endless uncharted territory, and it lets me live the dream of so many children: to be an explorer in wild and untamed countries. I love it because I can share it with others, and they can share it with me, even if we don't always understand each other. I love it because it is a universal language of thought and reason. I love it for the wonderful people who do math, research it and teach it.

"There are many other reasons, but these are the ones I can think of at the moment."

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